Optimum Spatially Constrained Turns for Agile Micro Aerial Vehicles

Aditya A. Paranjape*, Kevin Meier†, Soon-Jo Chung‡, and Seth Hutchinson§

University of Illinois at Urbana-Champaign, Urbana, IL 61801

The objective of this paper is to derive a rapid turn-around maneuver for micro aerial vehicles flying in densely cluttered environments, in the form of variable altitude turns optimized for spatial constraints. The design results in a motion primitive which can be implemented on miniature aircraft at large. The primitive is tested on an MAV equipped with just yaw control and no roll control. The maneuver design not only yields a motion primitive for a turn-around maneuver, but also sheds light on the aircraft design features that enable such a maneuver to be accomplished given some information about the obstacle field.

Nomenclature

\( C_L, C_D \) coefficients of lift and drag

\( m, S \) mass of aircraft and wing area

\( p, r \) aircraft roll rate and yaw rate, respectively

\( T \) thrust per unit mass

\( V \) flight speed

\( x, y, h \) global position coordinates, with \( h \) denoting the altitude

\( \alpha, \beta \) angle of attack, sideslip

\( \gamma, \chi \) flight path angle and heading angle

\( \mu \) wind axis roll angle

\( \rho \) density of air

\{ \} time derivative

I. Introduction

The objective of this paper is to describe turn around strategies for an agile micro aerial vehicle (MAV) which minimize the volume inside which the turn is accomplished. This maneuver is intended for MAV flight in the midst of densely clustered obstacles and with a limited field of sensing (e.g., using vision). In such scenarios, it is not difficult to imagine a motion planning algorithm failing to yield a feasible path, in which case an aggressive turn-around (ATA) maneuver provides a way to prevent an otherwise inevitable collision (see Fig. 1). The maneuver derived in this paper can be incorporated into a motion planning algorithm which guides an MAV through an obstacle field, as described in a companion paper.

Optimal control problems have been addressed by the flight mechanics community in the past fifty years, mainly in the form of minimum time to climb and turn. In contrast, the present paper works with a minimum

---

*Postdoctoral Research Associate, Department of Aerospace Engineering, and AIAA Member; Email: paranja2@illinois.edu
†Graduate student, Department of Electrical and Computer Engineering, and AIAA Student Member; Email: kcmeye2@illinois.edu
‡Assistant Professor, Department of Aerospace Engineering, and AIAA Senior Member; Email: sjchung@illinois.edu
§Professor, Electrical and Computer Engineering; Email: seth@illinois.edu
volume-to-turn formulation. A similar problem was attempted for tail sitter aircraft by Matsumoto and co-authors, although their problem formulation was confined to the longitudinal plane and required transient and risky stalling maneuvers. On the other hand, the solution proposed in this paper takes the form of a rapid variable-altitude turn, illustrated schematically in Fig. 2.

The equations of motion of a turning aircraft, notwithstanding the rigid body dynamics, consist of six states: the three position coordinates, the flight speed, flight path angle and the heading. The presence of six states with no obvious decoupling between them makes the analysis quite cumbersome, and several authors sought elegant approaches based on energy modeling and time scale separation. These approaches worked in part due to the fact that the authors considered fighter aircraft, whose size and speed present distinct time scales between motion in the longitudinal plane and turning, and partly due to the minimum time-to-turn formulation which allowed them to ignore the $x$ and $y$-position coordinates.

Micro aerial vehicles (MAVs), owing to their speed and size, present a unique problem in that the aforementioned time scale separation is almost non-existent. For instance, back-of-the-envelope calculations yield a phugoid time period between 1 and 5 seconds, which is not far removed from the turn dynamics. Moreover, the need to minimize the spatial volume of the turn implies that none of the aforementioned states can be ignored. This necessitates a numerical approach, although it may be possible to glean some insight into the solution from a knowledge of the physics.

In this paper, the rapid turn maneuver is first optimized numerically for just the kinematic equations.
of motion. The optimized solution yields an approximate optimum solution in the form of a control primitive wherein maximum angle of attack and roll angle are commanded with a time-delay between the two commands. The time-delay then becomes an optimization parameter matched to the constraints of the turning volume. Thereafter, we illustrate the feasibility of the ATA maneuver by designing a closed-loop controller for the full nonlinear model of an MAV equipped with just a vertical tail and no roll-control actuators. The closed-loop controller helps reproduce the maneuver obtained in the formal optimization of the kinematics-only model of the MAV.

The paper is organized as follows. The problem is formulated in Section II. The theoretical aspects of the optimization of the maneuver are discussed in Section III, and a numerical analysis for the special case of constant inputs is presented in Section IV. Section V lays the ground work for the control design problem to be addressed in the final version of the paper. Section VI concludes the paper.

II. Problem Formulation

We first ignore the rotational dynamics of the MAV. To simplify the notation, define

\[ k = \frac{\rho S}{2m}, \quad T = \frac{\text{Thrust}}{m} \] (1)

The aircraft dynamics are then described by the following equations:6

\[
\begin{align*}
\dot{x} &= V \cos \gamma \cos \chi, \quad \dot{y} = V \cos \gamma \sin \chi, \quad \dot{h} = V \sin \gamma \\
\dot{V} &= (T \cos \alpha - kV^2 C_D(\alpha)) - g \sin \gamma \\
\dot{\gamma} &= \left( \frac{T \sin \alpha}{V} + kV C_L(\alpha) \right) \cos \mu - g \frac{\cos \gamma}{V} \\
\dot{\chi} &= \left( \frac{T \sin \alpha}{V} + kV C_L(\alpha) \right) \sin \mu \cos \gamma
\end{align*}
\] (2)

We prescribe first-order dynamics for thrust \( T \), angle of attack \( \alpha \) and wind axis roll angle \( \mu \), with the understanding that a well-chosen first-order behavior can be achieved through sound control design:

\[
\begin{align*}
\dot{T} &= a_T(T_c - T), \quad \dot{\alpha} = a_\alpha(\alpha_c - \alpha), \\
\dot{\mu} &= a_\mu(\mu_c - \mu)
\end{align*}
\] (3)

where \( a_\cdot \)'s denote the time constants, and the subscript \( c \) denotes commanded values. The wind axis roll angle \( \mu \) differs from the body axis bank angle (also referred to as the body roll angle), denoted by \( \phi \), in that it is given by \( \sin \mu = \sin \phi \cos(\gamma + \alpha) \).

**Remark 1.** The equation for turn rate in (2) becomes singular with \( \gamma = \pi/2 \). The singularity can be tackled by replacing \( \chi \) with the unit vector along \( Z_w \triangleq T \sin \alpha + kV^2 C_L(\alpha) \) (where the subscript \( w \) refers to the wind frame). In fact, when \( \gamma = \pi/2 \), neither \( \chi \) nor \( \mu \) are well-defined; what is well-defined, however, are the \( x, y \) and \( z \) components of \( Z_w \). Therefore, the unit vector along \( Z_w \) can be used as a proxy for \( \chi \) and \( \mu \). This representation, with slightly different terminology, forms of the basis of the Frenet-Serret system. In the Frenet-Serret system, three vectors are used to define the body orientation: tangent to the flight path (along \( V \)), a normal vector to the flight path in a pre-defined plane (also referred to as torsion), and a bi-normal vector, the latter two directions coinciding with \( Z_w \).

Once the optimum \( T_c, \alpha_c \) and \( \mu_c \) are obtained, a control law is designed so that the aircraft maintains them throughout the maneuver. The key challenges here are: (1) the short period \( \alpha \) dynamics have a time period of 0.5 s, thereby limiting how well \( \alpha_c \) can be tracked; (2) the unstable lateral-directional dynamics pose the same challenge for \( \mu_c \) (which comes with the implicit requirement that \( \beta_c = 0 \)).

The control law is simulated in the MATLAB environment on a high fidelity model developed by the authors.3,10

III. Optimum Control Problem

The dynamics show a distinct hierarchy. The \( V \) and \( \gamma \) dynamics are tightly coupled: these are the classic phugoid dynamics. The dynamics of \( \chi \) (heading) and \( h \) (altitude) depend on \( V \) and \( \gamma \), while \( x \) and \( y \)
dynamics are driven by $V$, $\gamma$ and $\chi$. This does not necessarily imply a time-scale separation, as pointed out earlier. However, it does offer a route to reformulate the problem.

We start by ignoring the $x$ and $y$ dynamics. Instead, we define the turn radius $R$, which varies with $V$ and $\gamma$. We ignore $T\sin\alpha$, assuming that thrust is oriented along $V$. The turn radius is then given by

$$R \triangleq \frac{V \cos\gamma}{\dot{\chi}} = \frac{\cos^2\gamma}{kC_L \sin\mu}$$

Next, we estimate the duration of the maneuver. We start with the observation that $kVC_L = n_zg/V$, with $n_z$ denoting the load factor. The load factor is greater than 1, and its maximum value depends on $C_{L_{\text{max}}}$ and $V$. We will assume for the sake of argument that $n_z \sin\mu / \cos\gamma \approx 1$, a conservative bound for the ratio. Thus, $\dot{\chi} = kVC_L \sin\mu \cos\gamma \approx g/V \approx 2$. Therefore, a turn through 180° would last no longer than 1.5 s. It is best to assume a constant value of thrust $T$ in this short duration, given that thrust would ideally not change very quickly. Therefore, we conclude that our solution would involve constant values of $C_L = C_{L_{\text{max}}}$ and $T$ (whose value is to be determined). Finally, the control input $\mu$ can vary during the flight and can be treated as a control input for the maneuver. In general, it is safe to assume that a large value of $T$ would correspond to a maneuver with less constraints on $R$, while a tight constraint on $h$ would usually require a large value of $\mu$ with a smaller value of $T$.

A. Inverse Design

Equation 4 can also be used for inverse design. Suppose the obstacle field density requires a minimum value of $R$, denoted as $R_{\text{min}}$. The average value of $\cos^2\gamma$ can be assumed to be 1, while $C_{L_{\text{max}}} = 1$ is also a reasonable estimate. Typical values of $\mu$ are in excess of 30 deg (load factor of 2 during a level turn), and we may thus set $\sin\mu = 0.5$. Then, we need $k$ to satisfy

$$\frac{\rho S}{2m} \triangleq k \geq \frac{2}{R_{\text{min}}}, \text{ i.e., } \frac{m}{S} \leq \frac{R_{\text{min}}}{3}$$

The above expression automatically imposes an upper bound on the wing loading. The bound can be relaxed by computing $\cos\gamma$ more accurately, but that process requires a design to start with. Thus, it is possible to perform inverse design iteratively. An increased mass allowance can be used for installing improved sensing and computational capability on board. As a numerical illustration, $\gamma = 20$ deg permits an additional mass equal to 12% of the first estimate (formed by assuming $\gamma = 0$).

B. Maneuver Optimization

Aggressive turns are performed with the objective of reversing the aircraft heading, i.e., changing it by 180 deg, when collision-free forward flight is infeasible within the performance limitations of the aircraft. The word “aggressive” also suggests that these maneuvers take the aircraft to the boundary of its flight envelope, and they are unsustainable (and hence purely transient) in nature. The transient nature of the maneuver suggests that the control action required should use minimal control effort and rely instead on the physical characteristics of the aerial robot.

Mathematically, the design of an aggressive maneuver can be considered as an optimization problem:

$$\min_{\chi_i, \chi_f, \mu} f(x_{\text{max}}, y_{\text{max}}, h_{\text{max}})$$

subject to $|\chi_{\text{final}} - \chi_{\text{initial}}| = \pi$ (5)

where $x_{\text{max}}$, $y_{\text{max}}$, $h_{\text{max}}$ denote the maximum distances covered along the in the horizontal plane and maximum change in altitude, respectively, during the course of the turn. The function $f$ captures the spatial constraints: penalise $y_{\text{max}}$ heavily in a narrow corridor, and $x_{\text{max}}$ when an obstacle is straight ahead and near.

We start by estimating the duration of the maneuver. The duration is given by

$$\text{duration} = \frac{\pi}{\dot{\chi}_{\text{average}}} = \frac{\pi \cos\gamma}{kC_L \sin\mu}$$

The lift-to-weight ratio is referred to as the load factor $n_z$, so that $kVC_L = n_zg/V$. The load factor is greater than 1, and its maximum value depends on $C_{L_{\text{max}}}$ and $V$. We will assume for the sake of argument
that $n_z \sin \mu / \cos \gamma \approx 1.5$, a conservative bound for the ratio. Thus, $\chi = kV C_L \sin \mu / \cos \gamma \approx 1.5 \ g / V \approx 1.5$. Therefore, a turn through 180° would last approximately 2 s. It is best to assume a constant value of thrust $T$ in this short duration, given that thrust would ideally not change very quickly. In general, a large value of $T$ is required for a maneuver with tight constraints on $R$, while a tight constraint on $h$ require a large value of $\mu$ with a smaller value of $T$.

We design the ATA primitive systematically. First, we treat $T_c, \alpha_c$ as well $\mu_c$ as time-varying control inputs, which also represents the “ideal” case of perfectly characterised actuator dynamics, and zero time delay in sensing and actuation. We show, in particular, that the optimal angle of attack command $\alpha$ and constant and equal to the stall angle of attack $\alpha_{\text{stall}}$. We also argue that a constant values of thrust and wind axis roll angle can be commanded for an approximately optimum solution, while matching the time delay between the commands of $\alpha_c$ and $\mu_c$ to the volume available for turning. This design, incidentally, is identical to that used by human pilots.

We state the optimal control problem for the ATA as follows: compute the control inputs $T_c(t), \alpha_c(t)$ and $\mu_c(t)$ to minimize the cost

$$J = \eta_\gamma \gamma^2(t_f) + \int_0^{t_f} (\eta_x x^2 + \eta_y y^2 + \eta_h h^2 + \eta_T T^2_c + \eta_\mu \mu^2) dt$$

subject to the dynamics in Eq. (2) and (3), and with the terminal constraints $\chi(t_f) = \pi$ (heading change through 180 deg) and $\mu(t_f) = 0$ (the aircraft should have rolled out to level flight). The weights $\eta_x, \eta_y$ and $\eta_h$ are chosen to match the spatial constraints, while the control inputs $T_c, \alpha_c$ and $\mu_c$ are subject to the bounds $T_c \in [0, 1.2], |\alpha_c| \leq 50$ deg and $|\mu_c| \leq 60$ deg.

Define the Hamiltonian

$$H = \eta_x x^2 + \eta_y y^2 + \eta_h h^2 + \eta_T T^2_c + \eta_\mu \mu^2 + \lambda_x V \cos \gamma \cos \chi + \lambda_y V \cos \gamma \sin \chi + \lambda_h V \sin \gamma + \lambda_T V \cos \gamma \left( \frac{T \sin \alpha}{V} + kV C_L(\alpha) \right) \cos \mu - \frac{g \cos \gamma}{V}$$

$$+ \lambda_\gamma \left( \frac{T \sin \alpha}{V} + kV C_L(\alpha) \right) \frac{\sin \mu}{\cos \gamma} + \lambda_\mu a_\mu (\mu_c - \mu) + \lambda_T a_T (T_c - T) + \lambda_\alpha a_\alpha (\alpha_c - \alpha)$$

This gives us the following dynamical equations for the co-states

$$\dot{\lambda}_x = -2\eta_x x, \quad \dot{\lambda}_y = -2\eta_y y, \quad \dot{\lambda}_h = -2\eta_h h$$

$$\dot{\lambda}_V = -(\lambda_x \cos \gamma \cos \chi + \lambda_y \cos \gamma \sin \chi + \lambda_h \sin \gamma) + 2\lambda_V kV C_D + \lambda_\gamma \left( \frac{T \sin \alpha}{V^2} - kC_L \right) \cos \mu - \frac{g \cos \gamma}{V^2}$$

$$+ \lambda_\gamma \left( \frac{T \sin \alpha}{V} + kV C_L \right) \frac{\sin \mu}{\cos \gamma}$$

$$\dot{\lambda}_\gamma = \lambda_x V \sin \gamma \cos \chi + \lambda_y V \sin \gamma \sin \chi - \lambda_h V \cos \gamma) + \lambda_V V \cos \gamma - \frac{g \sin \gamma}{V}$$

$$- \lambda_\gamma \left( \frac{T \sin \alpha}{V} + kV C_L \right) \frac{\sin \mu \sin \gamma}{\cos \gamma}$$

$$\dot{\lambda}_\chi = \lambda_x V \cos \gamma \cos \chi - \lambda_y V \cos \gamma \cos \chi$$

$$\dot{\lambda}_\mu = a_\mu \lambda_\mu + \left( \frac{T \sin \alpha}{V} + kV C_L \right) \left( \lambda_\gamma \sin \mu - \lambda_\chi \frac{\cos \mu}{\cos \gamma} \right)$$

$$\dot{\lambda}_T = a_T \lambda_T - \dot{\lambda}_V \cos \alpha - \frac{\sin \alpha}{V} \left( \lambda_\gamma \cos \mu + \lambda_\chi \frac{\sin \mu}{\cos \gamma} \right),$$

$$\dot{\lambda}_\alpha = a_\alpha \lambda_\alpha + \lambda_V (T \sin \alpha + kV^2 C_D) - \left( \frac{T \cos \alpha}{V} + kV C_L \right) \left( \lambda_\gamma \cos \mu + \lambda_\chi \frac{\sin \mu}{\cos \gamma} \right)$$

The boundary conditions for the co-states are given by

$$\lambda_x(t_f) = \lambda_y(t_f) = \lambda_h(t_f) = \lambda_V(t_f) = \lambda_T(t_f) = \lambda_\alpha(t_f) = 0,$$

$$\lambda_\gamma(t_f) = 2\gamma(t_f)$$

$$H(t_f) = 0$$

(8)
The optimum control inputs are found by solving

\[
\frac{\partial H}{\partial T_c} = \frac{\partial H}{\partial \alpha_c} = \frac{\partial H}{\partial \mu_c} = 0
\]

which gives

\[
T_c = \frac{\lambda_T a_c}{\eta_T}, \quad \alpha_c = \frac{\lambda_\mu a_c}{\eta_\mu}, \quad \mu_c = -\alpha_{\text{stall}} \text{sign}(\lambda_\alpha)
\] (9)

In addition, we limit \(T_c\) and \(\mu_c\) to suitable limiting values.

We solve the two point boundary value problem in (2), (3), (7), (9) numerically using GPOPSII (which, incidentally, does not require the user to supply the co-state dynamics in Eq. (7)). Results for the two cases \([q_x q_y q_z]=[11.5]\) and \([15.1]\) are plotted in Figs. 3 and 4 respectively. These cases capture short and narrow volumes, respectively. In both cases, the aircraft performs a 3-D turn. The angle of attack reaches the stall value rapidly. The specific thrust is more or less constant, around 5 m/s\(^2\). Although the maximum value of \(\mu_c = 60\) deg is attained in both cases, the important distinction is the instant at which the roll is commenced with respect to the pull-up (which measures the time delay between the pull up to \(\alpha_{\text{stall}}\) and the roll to \(\mu_{c,\text{max}}\)). Note that, in both cases, \(\mu\) returns to zero after the aircraft has turned through 150 deg.

The pull up to \(\alpha_{\text{stall}}\) with wings more or less level (i.e., \(\mu_c = 0\)) causes the aircraft to climb and slow down. A larger time delay between the pull-up to \(\alpha_{\text{stall}}\) and the roll to \(\mu_{c,\text{max}}\) causes the aircraft to slow down considerably while gaining altitude, after which it changes heading rapidly before accelerating and descending to its previous altitude. On the other hand, a smaller time delay between the pull-up and the roll leads to a more or less steady turn, as is evident from a comparison of Figs. 3 and 4.

The above analysis leads to the hypothesis that aggressive turns can be performed by commanding constant values of \(T_c\), \(\alpha_c\) and \(\mu_c\), and tuning the time-delay between pull-up and the roll to ensure a collision-free turn in the available volume. The resulting ATA is described in Algorithm 1. The maneuver starts with a pull-up to \(\alpha = C_{L,\text{max}}^{-1}(C_L)\), followed by a roll to a prescribed value of \(\mu\). The parameter \(\tau_d\), which is the time delay between \(\alpha_c\) and \(\mu_c\) commands for the ATA, needs to be optimized. Note that, in a practical setting, the time delay would be computed online using a formula that uses information about the turning volume (available through sensing). The time-delay would correspond physically to the time lapsed between the actual transmission of the pull-up and roll commands.

Figure 5 depicts ATA trajectories for various values of \(\tau_d\), with \(\mu_{\text{max}} = 1.1\) rad. For example, \(\tau_d \approx 1\) s can be used to turn around in a long but narrow corridor with a minimum turn radius of less than 0.5 m, while \(\tau_d = 0\) can be used to turn when the turning volume has a sideways space of nearly 3 m but the permissible change in altitude is under 0.3 m. Interestingly, the latter case also minimizes the distance covered along the \(x\) axis (i.e., in the direction of the original flight path). The conclusions obtained here match those from Figs. 3 and 4.

Although the qualitative trends in Fig. 5 are independent of the initial conditions, a non-zero \(\gamma\) can improve the turning performance significantly. A lower initial speed reduces the forward distance covered during the turn, it has virtually no bearing on the actual turn radius.

For the ATA maneuver in Algorithm 1, we need to estimate \(\Delta \chi_{\text{crit}}\), the heading change after which the aircraft commences recovery to level flight. The value of \(\Delta \chi_{\text{crit}}\) depends on the agility; for an aircraft with infinite agility, we would set \(\Delta \chi_{\text{crit}} = 0\). For an aircraft with a finite agility, i.e., with \(\mu = a_\mu (\mu_c - \mu)\)

An approximate value of \(\Delta \chi_{\text{crit}}\) can be found by setting \(\mu_{\text{ATA}} \approx \pi/2\) and \(\cos \gamma \approx 1\):

\[
\Delta \chi_{\text{crit}} \approx \frac{T \sin \alpha/V + kV C_L}{a_\mu} \sin \mu_{\text{ATA}}
\]
Figure 3. Trajectory and flight parameters when $\eta_x = \eta_h = 1$ and $\eta_y = 5$.

Note that $C_L$ is set at the outset by fixing $\alpha$; $k$ and $a_\mu$ are known parameters, and $V$ can be measured. It may be possible to compute $\Delta \chi_{\text{crit}}$ continuously, and commence recovery when its value matches the remaining value of the heading change.

For the aircraft considered here, $k = 0.37$, $C_L = 1.6$ during the turn, $a_\mu = 8$, while $V \approx 5$ m/s during the recovery phase. The thrust was set to $T \approx 5$ for the simulations in Figs. 3 and 4. This gives $\Delta \chi_{\text{crit}} \approx 42$ deg, which is quite close to that obtained in Figs. 3 and 4.

To compute the thrust $T_c$ for the maneuver in Algorithm 1, we start by assuming a zero change in altitude and final speed, in which case energy balance implies that the role of thrust is to compensate for the energy dissipated due to the drag, so that

$$T_c = \frac{\int kC_D V^2 dl}{\int V dt} = \frac{kC_D \int_0^t V^2 dt}{\int_0^t V dt}$$

If we assume that the aircraft slows down almost to zero and the values of acceleration and deceleration are constant, we get

$$T_c = \frac{kC_D (\alpha_{\text{max}}) V_0^2}{2}$$

This value, however, needs to be used with caution because the aircraft need not recover all of its kinetic energy at the end of the turn (as seen in Figs. 3 and 4). This can reduce the thrust requirement significantly as seen in Figs. 3 and 4.
IV. Control Design Example

The rapid turning maneuver was optimized in the previous sections using a kinematics-only model. The assumption underlying that design was the provision of an inner loop controller capable of providing the desired control input profiles. In particular, as argued in the previous section, we set the commanded thrust $T_c$ to a constant value; the elevator is further set to a constant, set to its uppermost value, to ensure the highest possible angle of attack. The problem of ensuring the desired value of wind axis roll angle $\mu_c$ needs to be addressed on a case-by-case basis.

The problem of ensuring the desired value of $\mu_c$ is a combined roll-yaw control problem of ensuring the desired bank angle and zero sideslip. The MAV shown in Fig. 6 lacks dedicated roll control devices and has a rudder for yaw control. The rudder is controlled to ensure that the wind axis roll angle achieves the desired value $\mu_c$ using roll rate feedback, while the stability of the roll-yaw dynamics is used to ensure that the sideslip stays bounded (and within reasonable limits). A simple, two-step feedback controller is designed as follows:

$$
\begin{align*}
  p_c &= k_{p,\mu}(\mu_c - \mu) + k_{I,\mu} \int_0^t (\mu_c - \mu) \, dt \\
  \delta_r &= k_{p,\rho}(p_c - p) + k_{I,\rho} \int_0^t (p_c - p) \, dt
\end{align*}
$$

(11)

The choice of proportional-integral (PI) controllers was inspired by recent results\textsuperscript{10} which demonstrated that dynamic inversion-based control laws for nonlinear systems can be transformed \textit{exactly} to PI/PID laws together with guarantees on the stability of the closed-loop system. Figure 7 shows the 3-D trajectory, the
Figure 5. Plots showing the aggressive turn trajectory for $\tau_d \in [0, 1]$ (dark curves denote a larger time delay) in the $x-y$ and $x-z$ planes.

Algorithm 1 Agile turn-around (ATA) maneuver

Result: $\chi \leftarrow \chi \pm \pi$

Initialize $t \leftarrow t_0$ and $\chi_f = \chi \pm \pi$

while $\chi \neq \chi_f$ do

$\alpha_c \leftarrow \alpha_{\text{stall}}, T_c$ from (10)

if $t > t_0 + \tau_d$ and $|\chi_f - \chi| < \Delta \chi_{\text{crit}}$ then

$\mu_c \leftarrow \mu_{c, \text{max}}$

else

$\mu_c = 0$

end

end

Figure 6. Picture of MAV whose model is used for simulating a rapid turn.

wind axis roll angle and the flight speed. The time delay $\tau_d$ is set to 0.5 s. The turn diameter is seen to be 1.4 m and the wind axis roll angle and flight speed profiles are similar to those shown in Fig. 3 and which were obtained by formally optimizing the kinematics-only model.
Figure 7. Simulation of an ATA maneuver using the closed-loop controller in Eq. (11). In the 3-D plot, the initial and final positions are denoted by the red and green triangles, respectively.

Arknowledgements

This research was funded by the Office of Naval Research (ONR) under Award No. N00014-11-1-0088 and by the National Science Foundation (NSF) under Grant IIS-1253758. The second author was funded through a SMART Scholarship by OSD-T&E (Office of Secretary Defense-Test and Evaluation), Defense-Wide / PE0601120D8Z National Defense Education Program (NDEP) / BA-1, Basic Research.

V. Conclusion

This paper presented an optimal turning motion primitive called agile turn-around maneuver (ATA). In our companion paper [3], we incorporate this agile motion primitive to construct a motion planning algorithm
for an aircraft to navigate through a dense forest environment. The ATA primitive is parametrized by thrust and the wind axis roll angle whose optimal values are a function of the turning volume envelope. These optimal values were determined numerically, and led to the design of a simplified ATA primitive parametrized by the time delay between angle of attack and wind-axis roll angle commands. The maneuver was tested on an MAV simulator and the inner loop controller designed for the MAV was seen to reproduce the profile obtained as part of the optimization of the kinematics-only model.

References