Information-Driven Systems Engineering Study of a Formation Flying Demonstration Mission using Six CubeSats

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Small satellites are suitable for formation flying missions where a large number of spacecraft serve as distributed sensors for applications like synthetic aperture radar, interferometry, etc. A survey of existing or proposed small satellite missions concludes that there is a dearth of formation flying missions using four or more spacecraft that require formation maintenance. This paper presents a systems engineering based design of a formation flying technology demonstration mission that requires precise formation maintenance and reconfigurations and highlights the challenges that need to be overcome for its successful implementation. The goal of this paper is to provide directions for future research and development in spacecraft formation flying technologies.

I. Introduction

There have been significant improvements in the field of formation flying (FF) for small satellites. Formation flying is defined as a set of more than one spacecraft whose dynamic states are coupled through a common control. Multiple small satellites, flying in formation, can out-perform a single monolithic satellite in fields such as synthetic aperture radar, interferometry, etc. A survey of existing or proposed small satellite missions concludes that there is a dearth of formation flying missions using four or more spacecraft that require formation maintenance. However, a survey of existing or proposed CubeSat FF missions concludes that there is a dearth of FF missions using four or more CubeSats that require formation maintenance and reconfiguration maneuvers. Hence, the aim of this paper is to present a systems engineering based design of a formation flying technology demonstration mission using 4-6 CubeSats. The paper also highlights the
(1a) 4 CubeSats maintain a tetrahedron formation in Low Earth Orbit (LEO)

(1b) 6 CubeSats perform multiple optimal reconfiguration among different J₂-invariant orbits.

Figure 1: The overview diagrams of the two proposed FF technology demonstration missions.

various technological challenges that need to be overcome in order to successfully launch such a mission and provides directions for future research and development in spacecraft formation flying technologies for CubeSats.

Two different FF technology demonstration ideas are presented in this paper, one with active formation maintenance and the other one with passive formation maintenance. Fig. 1a shows the overview diagram for the first (active formation maintenance) proposed mission. In this FF mission, four 3U sized CubeSats maintain a tetrahedron geometry with a nominal inter-satellite distance of 50 m at an altitude of 400 km. The missions objective is to autonomously reconfigure to the desired geometry after launch and then maintain it while avoiding inter-satellite collisions. The schematic diagram of the second FF (passive formation maintenance) mission is shown in Fig. 1b. This mission begins with 6 CubeSats reconfiguring into J₂ invariant relative orbits. It has been shown that once a group of satellites enter a J₂ invariant relative orbit, they require very minimal amounts of fuel to maintain that collision free orbit. In this mission, after periodic intervals, the CubeSats will reconfigure into a different J₂ invariant relative orbit and maintain that orbit and so on. However, once a group of CubeSats are in an J₂ invariant relative orbit, they will not hold constant relative positions, they will move such that they do not collide and return to their positions periodically.

The aim behind the two missions depicted in Fig. 1a and Fig. 1b is very different. The tetrahedron formation configuration can be used for missions which require the CubeSats to hold the position constantly, like interferometry, synthetic aperture radar etc. The amount of fuel consumed in this mission is very high because the CubeSats are being forced to maintain non-Keplerian orbits and hence, the lifetime of the mission tends to be short (shown in Section IV). In the multiple reconfiguration mission (Fig. 1b), the formation would look the same after constant time intervals. Hence, it can be used for missions such as sampling the ionosphere, in which images would be taken after constants instants of time. Since no active formation maintenance is required in this mission, i.e., the CubeSats do not have to be constantly firing their thrusters to maintain their position, the amount of fuel required for this mission is pretty low and hence it can be used for typically longer missions (shown in Section IV).

The breakdown of this paper is as follows. State-of-the-art mission level requirements are identified in Section II. Section III identifies the list of possible sensors and actuators that can be used in each CubeSat and that satisfy the requirements that have been defined in Section II. It also identifies the technologies that are the major bottlenecks for the design of formation flying missions. Section IV gives a detailed analysis of the controls subsystem for both the tetrahedron formation mission, as well as the multiple reconfiguration mission. Relative sensing studies using differential GPS has been shown in Section V. We conclude the paper in Section VI.
II. Requirements Analysis

In this section, the mission-level requirements, defined by pushing the state of the art in CubeSat FF, will be broken down into subsystem requirements to identify subsystems that need to be further developed before flight. The proposed mission uses 3U CubeSats, which have certain design limitations to fit within the CubeSat standard. A 1U CubeSat is $10 \times 10 \times 10$ cm with a mass of 1.33 kg. A 3U CubeSat must be a $10 \times 10 \times 34$ cm rectangular box with a maximum mass of 4 kg. An optional 36 mm addition can be added as shown in Fig. 2 to accommodate more propulsion options. The batteries used may not store above 100 Watt-hours and the RF output cannot exceed 1.5 W. There are many more requirements, but they pertain more to the actual operation and software design of the satellites and are not necessary for the purpose of this discussion. A more thorough description can be found in the CubeSat Design Specification.\(^8\)

![3U CubeSat](image)

Figure 2: Schematic diagram of a 3U CubeSat (image credit: California Polytechnic State University\(^8\)).

The requirements are mainly defined by the mission itself. Keeping four satellites in a tight formation for at least 100 orbits sets the required $\Delta V$, but this is dependent on the formation size, altitude of the orbit, formation shape, and other characteristics that will be discussed below in the controls section (Section IV). The mission is also intended to be state of the art, to use the best possible sensors and actuators that satisfy the mission requirements. This drives the requirements in every other aspect of the mission. For the purpose of this systems engineering design, the following subsystems are considered: on-board computing, controls, power, communications, structures and thermals.

A. Functional Dependencies

When considering the requirements of the various subsystems as defined above, it is crucial to consider how the subsystems interrelate. These dependencies are illustrated in the Fig. 3.

![Functional dependencies](image)

Figure 3: Functional dependencies of the subsystems. The arrows go from the subsystem which requires the data to the subsystem which provides that data and arrows of the same color belong to the same subsystem.
On-board computing (OBC) needs temperature data to monitor the health of the components, ground station commands from communications, power to run its components, and state data to process for the controls subsystem. Controls subsystem needs state data from the other satellites through communications, data processed by the OBC, and power to run its components. Power subsystem needs to know what components should be running from the OBC, and needs the controls subsystem to maintain a pointing which allows solar arrays to function (if they are the power generation type selected). Communications subsystem needs to send and receive state data between the controls subsystem and other satellites, commands from the OBC, and power for its components. Thermals subsystem needs temperature data from all subsystems with active electronics. A point to note is that the structures subsystem is assumed static, unchanged after launch so it does not take any data other than thermal.

B. State of the Art

The main goal of this mission is to reach state of the art of current FF CubeSats. From the review of CubeSat FF missions,\(^6\) the state of the art for position and attitude determination and control is given in Table 1. The communications and on-board computing requirements are chosen to support the position and attitude needs. The state of the art is defined mostly from the Canadian Advanced Nanospace eXperiment-4+5 (Can-X)\(^9\) and Cubesat Proximity Operations Demonstration (CPOD)\(^10\) missions. Both of these missions involve two satellites, Can-X maintains a strict inter-satellite distance between two satellites approximately 8U in size and the second attempts a dock between two 3U satellites. While Can-X is slightly larger than the 3U satellite discussed for this mission, the main innovation is in the differential GPS (DGPS) algorithm on-board, which allows the mission to achieve such remarkable relative position determination. Can-X has not yet flown, so the DGPS scheme is not yet proven.

Table 1: State of the art for position and attitude sensors and actuators. The \(\Delta V\) required is calculated in Section IV

<table>
<thead>
<tr>
<th>Metric</th>
<th>Proposed Missions</th>
<th>Achievable Today</th>
<th>Achievable 2-years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute</td>
<td>Can-X(^9)</td>
<td>1.2 m (RMS)</td>
</tr>
<tr>
<td>Determination</td>
<td>Relative</td>
<td>2.5 cm (RMS)</td>
<td>&lt; 2.4 m</td>
</tr>
<tr>
<td></td>
<td>2.5 cm (RMS)</td>
<td>Can-X(^9)</td>
<td>1.2 m (RMS)</td>
</tr>
<tr>
<td></td>
<td>1 m</td>
<td>Can-X(^9)</td>
<td>5 m ((\Delta V/)orbit - 0.93 m/s)</td>
</tr>
<tr>
<td></td>
<td>0.007 deg</td>
<td>CPOD(^10)</td>
<td>&lt;0.007 deg</td>
</tr>
<tr>
<td>Attitude</td>
<td>Relative</td>
<td>0.5 deg</td>
<td>&lt;0.014 deg</td>
</tr>
<tr>
<td>Determination</td>
<td>0.014 deg</td>
<td>Can-X(^9)</td>
<td>0.021 deg (3(\sigma))</td>
</tr>
<tr>
<td></td>
<td>0.042 deg</td>
<td>Can-X(^9)</td>
<td>0.042 deg (3(\sigma))</td>
</tr>
</tbody>
</table>

III. Possible Sensors and Actuators

After a thorough review of current and near-ready technologies applicable to the studied subsystems, various key technologies that could satisfy the above requirements were identified and separated into two categories (technologies that can be immediately implemented and technologies that can be implemented in approximately two years). Emphasis was placed on commercial-off-the-shelf (COTS) products that could be easily obtained and implemented. Overall the total mass, size, and power of the components are major concerns as well as the subsystem-specific performance criteria. Since a plethora of more than adequate options exist to meet on-board computing and power needs, options for these subsystems will not be discussed in detail.

A. On-Board Computing

This subsystem error checks the components, sends, stores, and receives data, and performs the on-board computations. The requirements for this subsystem will largely be driven by the computational demands of the control algorithm, which is also dependent on the mission parameters. The data storage required is also
dependent on the communications system. For example, a satellite with a low communication frequency will need more data storage because of the lag between downlinks. The available options, listed in Table 2, are very similar. All of the presented options have a TRL of ≥8.

### Table 2: OBC options.

<table>
<thead>
<tr>
<th>Name</th>
<th>RAM</th>
<th>Clock Speed</th>
<th>Size</th>
<th>Power</th>
<th>Voltage</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>NanoMind A712D</td>
<td>2MB Static</td>
<td>8 - 40 MHz</td>
<td>96 mm × 90 mm × 10 mm</td>
<td>-</td>
<td>3.3 V</td>
<td>50 - 55 g</td>
</tr>
<tr>
<td>Cube Computer 12</td>
<td>256kb EEPROM</td>
<td>4 - 48 MHz</td>
<td>96 mm × 90 mm × 10 mm</td>
<td>&lt; 200 mW</td>
<td>3.3 V</td>
<td>50 - 70 g</td>
</tr>
<tr>
<td>Q6</td>
<td>2*128 MB LPDDR</td>
<td>-</td>
<td>78 mm × 38 mm × 19 mm</td>
<td>1 W (typical)</td>
<td>3.3 V to 5.5 V</td>
<td>23 g</td>
</tr>
<tr>
<td>Andrews 150</td>
<td>512kb SD</td>
<td>150 MHz</td>
<td>15 mm × 97 mm × 90 mm</td>
<td>&lt; 1 W (nominal), 3 W (maximum)</td>
<td>6.5 V, 12 V, 28 V</td>
<td>70 g</td>
</tr>
<tr>
<td>Andrews 160</td>
<td>64 MB SD</td>
<td>400 or 100 MHz</td>
<td>15 mm × 97 mm × 90 mm</td>
<td>&lt; 5 W (nominal), 9 W (maximum)</td>
<td>6.5 V, 12 V, 28 V</td>
<td>70 g</td>
</tr>
</tbody>
</table>

B. Position and Attitude Determination and Control

This subsystem determines the satellite’s relative state and uses that information along with the desired state to control the position and attitude. It is emphasized that relative position control takes precedence over individual satellite orbit control in FF. The controls requirements are derived from the inter-satellite position and attitude accuracy required by the mission.

1. Attitude Determination

Attitude is extremely important in formation flying missions because many CubeSat science missions require maintained pointing, especially imaging-based missions. CubeSat thrusters are not gimbaled, so accurate attitude control is needed to ensure the proper thrust vector. Accurate attitude determination is even more important, actuators can usually only control to about ten times the knowledge accuracy. Several different attitude sensors are presented in Table 3 and Fig. 4. The most accurate are the star trackers, which compare the observed star field to a known map of the stars to determine attitude and can reach a precision as low as 7-24 arcseconds. However, as the figure shows, the TRL for CubeSat star trackers is about six or seven, which is below that of other technologies. Several of the other options operate using the same principle, like sun sensors and nadir sensors, which determine attitude based off of the location of the sun and Earth nadir respectively. These are around two orders of magnitude less accurate because the star trackers compare multiple points across the entire field of view where sun and nadir sensors have a relatively small target, but they have been proven in space and have options at a TRL of 9. The final option is magnetometers, which compare magnetic field measurements to what is known about the Earth’s magnetic field. These usually only resolve one or two axes of the attitude and can only achieve accuracies of around 5°, so they are not generally suitable for missions with stringent attitude constraints.
Figure 4: Comparison of applicable attitude sensors. Numbering of the bubbles is as defined in Table 3.

Table 3: Attitude sensor options.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Accuracy</th>
<th>Type</th>
<th>TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Blue Canyon Tech. Nano Star Tracker</td>
<td>7-24 arcsec</td>
<td>Star Tracker</td>
<td>≥ 6</td>
</tr>
<tr>
<td>2</td>
<td>Melexis MLX90615</td>
<td>0.5 deg</td>
<td>Earth Nadir Sensor</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>Honeywell HM C6042</td>
<td>0.15 mG</td>
<td>2-Axis Magnetometer</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>Honeywell HM C1041Z</td>
<td>0.15 mG</td>
<td>1-Axis Magnetometer</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>Space Micro Coarse Sun Sensor</td>
<td>5 deg (1 axis)</td>
<td>Sun Sensor</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>Space Micro Medium Sun Sensor</td>
<td>1 deg (2 axis)</td>
<td>Sun Sensor</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>Berlin Space Technologies ST-200</td>
<td>30-200 arcsec</td>
<td>Star Tracker</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>Digital Fine Sun Sensor CubeSat Shop</td>
<td>0.1 deg</td>
<td>Sun Sensor</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>CubeSat Sun Sensor</td>
<td>&lt; 0.5 deg</td>
<td>Sun Sensor</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>MAI-100/200 Series</td>
<td>&lt; 1 deg</td>
<td>Integrated Package</td>
<td>≥ 7</td>
</tr>
<tr>
<td>11</td>
<td>MAI-400 Series</td>
<td>&lt; 1 deg</td>
<td>Integrated Package</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>Blue Canyon Tech. XACT</td>
<td>&lt; 0.003 deg</td>
<td>Integrated Package</td>
<td>&gt; 6</td>
</tr>
</tbody>
</table>

2. Attitude Control

The most widely available attitude control method for CubeSats is miniaturized reaction wheels. Reaction wheels can achieve up to two mN·m of torque with a small form factor. A few missions have also used aerodynamic wings, where the sides of the satellite open up to control the area that the atmospheric drag is acting upon. This is not suitable for all missions though, because it uses atmospheric drag so the attitude control maneuvers happen relatively slowly. The most interesting development in the field comes in the integrated packages. These packages typically contain reaction wheels, gyros, and some type of attitude sensor. The data from the sensor and the gyros are processed internally, which reduces the load on the on-board computer and ensures the attitude control code is reliable. With these benefits in mind, the best option of those presented in Table 4 is the Blue Canyon Technologies XACT Integrated Control Package for CubeSats. It claims to achieve pointing control of 0.003 to 0.007 degrees, which is an order of magnitude better than the other options.
Table 4: Attitude Actuators and Integrated Packages.

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Size</th>
<th>Type</th>
<th>Torque</th>
<th>TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aerocube 4 Retractable Wings(^{30})</td>
<td>2 wings, each 9x10 cm</td>
<td>Deployable Wings</td>
<td>N/A</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>BCT Micro Reaction Wheel(^{31})</td>
<td>43 × 43 × 18 mm</td>
<td>Reaction Wheel</td>
<td>0.6 mNm</td>
<td>N/A</td>
</tr>
<tr>
<td>3</td>
<td>BCT Integrated Attitude Control for CubeSats (XACT)(^{28})</td>
<td>0.5 U</td>
<td>Integrated Package</td>
<td>N/A</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>Sinclair Interplanetary RW - 0.007-4(^{32})</td>
<td>50 × 40 × 27 mm</td>
<td>Reaction Wheel</td>
<td>1 mNm</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>Sinclair Interplanetary RW - 0.01-4(^{33})</td>
<td>50 × 50 × 30 mm</td>
<td>Reaction Wheel</td>
<td>1 mNm</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>Sinclair Interplanetary RW - 0.03-4(^{29})</td>
<td>50 × 50 × 40 mm</td>
<td>Reaction Wheel</td>
<td>2 mNm</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>Berlin Space Tech. iACDS-100(^{34})</td>
<td>95 × 90 × 32 mm</td>
<td>Integrated Package</td>
<td>0.087 mNm</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>MAI-400 ADACS(^{27})</td>
<td>0.5 U</td>
<td>Integrated Package</td>
<td>0.625 mNm</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>MAI-300 Single Axis Reaction Wheel(^{35})</td>
<td>68.5 × 68.5 × 33 mm</td>
<td>Reaction Wheel</td>
<td>0.6 mNm</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>MAI-201 Miniature 3-Axis Reaction Wheel(^{36})</td>
<td>76.2 × 76.2 × 70 mm</td>
<td>Reaction Wheel</td>
<td>0.6 mNm</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>MAI-200 ADACS(^{37})</td>
<td>0.788 U</td>
<td>Integrated Package</td>
<td>0.6 mNm</td>
<td>7</td>
</tr>
</tbody>
</table>

3. Position Determination

In a FF CubeSat mission, effective relative position determination and control is of paramount importance to mission success. The science application determines the size of the formation, shape of the formation, as well as the accuracy of the control required. Formation flying missions extends the capabilities of CubeSats and increases the amount of science that can be obtained. The technologies that can be used are limited by the CubeSat requirements set forth by Cal Poly. A literature review of the formation flying CubeSat missions has concluded a relative determination on the centimeter scale was required. Additionally, the distance between...
the agents was assumed to be at least 30 meters. Various key technologies that could satisfy this requirement were identified and separated into two categories (technologies that can be immediately implemented and technologies that can be implemented in 2 years). Emphasis was placed on commercial off the shelf (COTS) products that could be easily obtained and implemented. The technologies that were initially considered for relative sensing include: microwave ranging, GPS, camera imaging, laser diode ranging, computer vision, infrared, and RF ranging. Based on specifications provided by manufacturers of these technologies, Fig. 6 shows the maximum range and sensing accuracy for each method. The ideal range is the top left region in Fig. 6. However, it is clear that GPS, Laser Diode Ranging, and RF Ranging are the only methods that meet the requirements of centimeter-level sensing accuracy and can operate at an inter-satellite distance of at least 30 m. It is important to note that the figure is only representative of a survey of products, and is not exhaustive.

![Figure 6: Comparison of relative position determination technologies.](image)

Relative position measurements provided by laser methods are promising, however maximum range and accuracy both scale with power input and overall size. Because of the CubeSat power and volume constraints, laser ranging was no longer considered for implementation in the near future based on these requirements. The remaining GPS and RF ranging options were further analyzed as potential candidates as the sensing method for a formation flying mission.

As shown in Fig. 6, COTS GPS receivers typically have accuracies of between 1 m to greater than 100 m. This range is dependent on the geometry of the satellites in view and other environmental disturbances. Centimeter level accuracies can be achieved through post processing of Differential GPS (DGPS) data. GPS receivers are available from a wide range of vendors and have relatively small form factors, which makes them attractive for a formation flying CubeSat mission. In order for GPS to be used on a formation flying mission, there must also be an inter-satellite communication system to share the ranging data. Additionally, a single GPS receiver onboard the satellite can be used for both absolute and relative position determination. The receivers considered for the trade study are shown below.

Because none of the options provide the positional accuracy needed directly, an additional algorithm must be developed to improve the accuracy to the centimeter level (see section V). Also, many COTS receivers that can be readily purchased are not space-rated, or tested for space applications, so the TRL was determined to be at least 6.

In addition to post processing GPS data for relative position determination in a formation flying mission, RF ranging was also. The maturity of this technology is not as high as that of GPS methods (approximately TRL 4-5), but can be implemented in 2 years. For this method, the satellite sends out RF signals to a target satellite. Based on the delay in the signal arrival, an inter-satellite distance can be determined. Since this method emits RF signals, the inter-satellite communication and relative position determination subsystems potentially can be combined to save power and space. Moreover, hardware has been specifically designed for formation flying missions by Swift Technologies to complete the task of inter-satellite communications and relative position determination. The maximum range is dependent on the power input and unit size. The
Table 5: GPS options.

<table>
<thead>
<tr>
<th>Name</th>
<th>Manufacturer</th>
<th>Frequency</th>
<th>Volume</th>
<th>Power (W)</th>
<th>Weight (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OEM615</td>
<td>NovAtel</td>
<td>Dual (L1,L2)</td>
<td>71 × 46 × 11 mm</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>SGR-05U</td>
<td>Surrey Satellite Technology</td>
<td>L1</td>
<td>70 × 45 × 10 mm</td>
<td>0.8 (@ 5V)</td>
<td>40</td>
</tr>
<tr>
<td>GPS-12-V1</td>
<td>SpaceQuest</td>
<td>L1</td>
<td>100 × 70 × 25 mm</td>
<td>1</td>
<td>&lt; 200</td>
</tr>
<tr>
<td>DFRSD GPS</td>
<td>Austin Satellite Design</td>
<td>Dual (L1,L2)</td>
<td>83 × 96 × 38 mm</td>
<td>4.7</td>
<td>350</td>
</tr>
<tr>
<td>SSBV GPS</td>
<td>SSBV Aerospace + Tech Group</td>
<td>L1</td>
<td>50 × 20 × 5 mm</td>
<td>&lt; 1</td>
<td>&lt; 30</td>
</tr>
</tbody>
</table>

accuracy and hardware is promising, but it is not yet widely available and thus only considered for future implementation.44

Based upon the requirements set for the mission, trade studies concluded that COTS hardware could not provide the required accuracy. For immediate implementation, DGPS was selected as it is readily available, and its accuracy can potentially be improved to centimeter level. For future implementation, RF ranging was selected for its small form factor and high accuracy.

4. Position Control

The ability to accurately control the position is the culmination of the efforts of all other subsystems. Without an adequate position control system, precise formation control is not possible. Unfortunately, the miniaturization of propulsion technologies has not progressed quite as quickly as other fields. Currently, most CubeSat thrusters use cold gas or hypergolic propellants, but CubeSat-sized electric propulsion thrusters are beginning to become available. The specific impulse ($I_{sp}$) of a thruster is proportional to the exit velocity of the flow and is useful to compare the propellant efficiency of the engines. A higher $I_{sp}$ engine burns propellant slowly, but typically has a lower thrust. Another useful quantity in propulsion system comparison is the amount of velocity change it can impart over the mission, or the $\Delta V$. The higher the $\Delta V$ of the system, the longer overall mission will last. Figure 7 shows that very few thrusters are available at high TRLs, and of the ones that are available, the $\Delta V$ achievable is not very high. There are some very promising options coming up in the next couple years, like the IL-FEEP thruster by Alta Space, which is stated to achieve 500 m/s with reasonable power and size and a very high $I_{sp}$. The list of thrusters is given in Table 6.

![Figure 7: Comparison of applicable thrusters. Numbering of the bubbles is as defined in Table 6.](image)
### Table 6: Thruster options

<table>
<thead>
<tr>
<th>No.</th>
<th>Name</th>
<th>Thrust (mN)</th>
<th>Volume (U)</th>
<th>∆V (m/s)</th>
<th>Power (W)</th>
<th>I&lt;sub&gt;sp&lt;/sub&gt; (s)</th>
<th>Propellant</th>
<th>TRL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Micro-PPT (Busek)&lt;sup&gt;45&lt;/sup&gt;</td>
<td>0.5</td>
<td>&lt;0.5</td>
<td>63</td>
<td>2</td>
<td>800</td>
<td>Teflon</td>
<td>&gt;5</td>
</tr>
<tr>
<td>2</td>
<td>MiPS (Vacco)&lt;sup&gt;46&lt;/sup&gt;</td>
<td>40</td>
<td>0.25</td>
<td>34</td>
<td>&lt;10</td>
<td>50</td>
<td>Isobutane</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Nanosatellite Micropropulsion System (ISIS)&lt;sup&gt;47&lt;/sup&gt;</td>
<td>0.1-10</td>
<td>1</td>
<td>100</td>
<td>2</td>
<td>50-100</td>
<td>Cold Gas</td>
<td>&gt;5</td>
</tr>
<tr>
<td>4</td>
<td>CHAMPS MPS-142 (Aerojet)&lt;sup&gt;48&lt;/sup&gt;</td>
<td>150</td>
<td>1</td>
<td>100</td>
<td>1</td>
<td>266</td>
<td>Hydrazine</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>IL-FEEP (Alta-Space)&lt;sup&gt;49&lt;/sup&gt;</td>
<td>1</td>
<td>1</td>
<td>500</td>
<td>3-8</td>
<td>2000</td>
<td>Ionic Cesium</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>Pressure-fed Electrospray Thruster (Busek)&lt;sup&gt;50&lt;/sup&gt;</td>
<td>0.7</td>
<td>0.56</td>
<td>151</td>
<td>&lt;9</td>
<td>800</td>
<td>Ionic Liquid</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>RF Ion BRFIT-1 cm (Busek)&lt;sup&gt;51&lt;/sup&gt;</td>
<td>0.67</td>
<td>1.25</td>
<td>244</td>
<td>10</td>
<td>1800</td>
<td>Xenon</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>MCD Propulsion Unit for CubeSats (Vacco)&lt;sup&gt;52&lt;/sup&gt;</td>
<td>5.4</td>
<td>1</td>
<td>167</td>
<td>15</td>
<td>70</td>
<td>EP-13</td>
<td>&gt;3</td>
</tr>
<tr>
<td>9</td>
<td>HYDROS (Tethers Unlimited)&lt;sup&gt;53&lt;/sup&gt;</td>
<td>800</td>
<td>0.5 or 1</td>
<td>300</td>
<td>0.5-10</td>
<td>300</td>
<td>Water</td>
<td>&gt;4</td>
</tr>
<tr>
<td>10</td>
<td>CHIPS (for 1U) (Vacco)&lt;sup&gt;54&lt;/sup&gt;</td>
<td>50</td>
<td>1.5</td>
<td>&gt;100</td>
<td>-</td>
<td>50-400</td>
<td>EP-76</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>Green Monoprop (Busek)&lt;sup&gt;55&lt;/sup&gt;</td>
<td>500</td>
<td>0.5</td>
<td>&gt;90</td>
<td>15</td>
<td>230</td>
<td>AF-M315E</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>RF Ion BRFIT-3cm (Busek)&lt;sup&gt;56&lt;/sup&gt;</td>
<td>2</td>
<td>1.25</td>
<td>4000</td>
<td>90</td>
<td>2500</td>
<td>Xenon</td>
<td>5</td>
</tr>
<tr>
<td>13</td>
<td>Micro Resistojet (Busek)&lt;sup&gt;57&lt;/sup&gt;</td>
<td>2-10</td>
<td>1</td>
<td>60</td>
<td>3-15</td>
<td>150</td>
<td>Ammonia</td>
<td>5</td>
</tr>
</tbody>
</table>

C. Power

The power subsystem must be able to generate, store, and distribute power to the components that need it. The amount of power required at different phases of the mission is difficult to determine without first selecting the components. In particular, the thrusters vary drastically (by type) in how much power they require, and so the thruster selection is an important driving factor in the power system design. The main options available for CubeSat power systems are solar arrays and batteries. CubeSat-sized solar arrays are limited in the amount of power available at a time, while batteries alone limit the overall mission duration. Representative solar array options are shown in Table 7.
Table 7: Power options

<table>
<thead>
<tr>
<th>Name</th>
<th>Height (mm)</th>
<th>Mass (g)</th>
<th>Power (W)</th>
<th>Voltages (V)</th>
<th>Regulated current (A)</th>
<th>Operating Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3U CubeSat EPS (Clyde Space)</td>
<td>15.3</td>
<td>83</td>
<td>1 to 50</td>
<td>3.3/5/12/Custom</td>
<td>4</td>
<td>-40 to 85</td>
</tr>
<tr>
<td>NanoPower P31u (GOMspace)</td>
<td>6.2 to 6.8</td>
<td>90</td>
<td>1 to 30</td>
<td>3.3 (Bus 1)/5 (Bus 2)</td>
<td>6 (Charge)/12 (Discharge)</td>
<td>-40 to 85</td>
</tr>
</tbody>
</table>

D. Communications

Inter-satellite communication is vital to FF missions to relay goals and decisions, verify gathered inter-satellite position and attitude data, and detect failures. Nearly all formation flying missions require some level of inter-satellite communication. The ideal communication subsystem would support inter-satellite communication with minimal lag using minimal power. Because of the size restrictions on the CubeSat, there is usually not enough space for a separate ground link antenna so the inter-satellite communications system must also be able to communicate with the ground, as well as between satellites. If the mission is designed to collect and downlink large amounts of data, the antenna and the frequency chosen will need to accommodate. Preliminary antenna selection can be made based off of previous CubeSat missions because the difference in cost, power, and size is not appreciable compared to the whole. Because of the size restriction, most CubeSat missions use UHF and S-band antennas because they are commercially available and can be contained within a CubeSat. Some research is being conducted on deployable antennas, so it is possible that more options will become available in the next two years. Of the options presented in Table 8, the XBee antenna has the highest transmitted power, no pointing requirements, a small size, and a high sensitivity.

Table 8: Inter-satellite communication options

<table>
<thead>
<tr>
<th>Name</th>
<th>Manufacturer</th>
<th>Frequency</th>
<th>Size</th>
<th>Transmit Power</th>
<th>Sensitivity</th>
<th>Antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>XBee 802.15.40</td>
<td>Digi International</td>
<td>2.4 GHz</td>
<td>24.38 × 32.94 × 3 mm</td>
<td>20 dBm</td>
<td>-106 dBm</td>
<td>Integrated Whip Chip</td>
</tr>
<tr>
<td>ZigBit 2.4 GHz Module</td>
<td>Atmel</td>
<td>2.4 GHz</td>
<td>24 × 13.5 × 2 mm</td>
<td>-17 to 3 dBm</td>
<td>-101 dBm</td>
<td>Balanced Dual Chip Antenna</td>
</tr>
<tr>
<td>EMB-Z2530PA</td>
<td>Embit</td>
<td>2.4 GHz</td>
<td>29 × 22 × 3 mm</td>
<td>20 dBm</td>
<td>-100 dBm</td>
<td>PCB Antenna</td>
</tr>
<tr>
<td>deRFmega256-23M12</td>
<td>Dresden Elektronik</td>
<td>2.4 GHz</td>
<td>31.5 × 13.2 × 3 mm</td>
<td>3 dBm</td>
<td>-100 dBm</td>
<td>RF pads</td>
</tr>
<tr>
<td>ZMN2430HP-R</td>
<td>RF Monolithics</td>
<td>2.4 GHz</td>
<td>30.48 × 25 × 3 mm</td>
<td>17 dBm</td>
<td>-95 dBm</td>
<td>RFIO pads</td>
</tr>
<tr>
<td>SWIFT - RelNav</td>
<td>Tethers Unlimited</td>
<td>S Band</td>
<td>100 × 100 × 25 mm</td>
<td>40 dBm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
E. Structures and Thermal

The structure of the satellites must support all of the components and minimize vibration during launch and thruster firing for the duration of the mission with minimum structural mass. The thermal subsystem monitors the temperatures of the various components and return to the acceptable range if it deviates. Depending on the selected orbit, an active heating/cooling system may be necessary. These subsystems are not crucial to this GNC-focused design and are left to future work.

F. Challenges

By comparing the requirements imposed by the mission to what is achievable using the sensors and actuators found in the survey has determined that most subsystems are adequately developed to suit the mission need. However, the position control (thrusters) and the relative position determination options cannot yet meet the requirements. The remainder of the paper will focus on characterizing and advancing what can be achieved in these two areas with currently available or emerging technologies.

IV. Control System Design

Since the controls subsystem was identified as a bottleneck for the required mission, we performed a detailed control system design. To define all the variables and the formations that we looked at, two coordinate frames have to be defined (see Fig. 8). The Earth-centered inertial (ECI) frame is used to describe the location of the chief CubeSat or some reference orbit and is called the chief orbit. The origin of the coordinate system is fixed at the center of mass of the Earth. The \( \hat{X} \) axis is aligned with the vernal equinox, the \( \hat{Z} \) axis is aligned towards the Earth’s spin axis or the north pole and the \( \hat{Y} \) is perpendicular to the other two according to the right handed coordinate system. It is important to note that there need not be an actual CubeSat at the location of the chief orbit, there can be a virtual leader, i.e., all other CubeSats will be distributed about that chief orbit but there will be no CubeSat on the chief orbit. The local-vertical/local-horizontal frame (LVLH) is centered on the chief CubeSat. Its \( \hat{x} \) or radial axis is aligned with the position vector pointing outwards from the center of the earth, the \( \hat{z} \) or cross-track axis is aligned with the angular momentum vector and the \( \hat{y} \) or along-track axis completes the coordinate system according to the right-hand convention. The LVLH frame is rotating with a rotating rate of \( \omega_z \) about the cross-track axis and \( \omega_x \) about the radial axis.

![Figure 8: ECI (\( \hat{X}, \hat{Y}, \hat{Z} \)) and LVLH (\( \hat{x}, \hat{y}, \hat{z} \)) frames](image)

The simulations and results associated with the two different FF technology demonstration missions (as described in Fig. 1a and Fig. 1b) are given in the following sections. Since the aim of our mission is a technology demonstration, some of the numbers that we have chosen might not be perfectly applicable to any particular scientific mission. The aim of the simulations was to develop an overall intuition for the kind
of numbers we obtain for such a mission and hence more emphasis has been placed on the general trends that the outputs show and range of their values, rather than any particular value.

A. Tetrahedron Formation Hold

The CubeSats initially launched from the Poly-PicoSatellite Orbital Deployer (P-POD), are assumed to be in a string-of-pearls formation with an inter-satellite separation of 1 km. The formation is then reconfigured to a regular tetrahedron of size 50 meters in the LVLH frame as shown in Fig. 9 and then maintained in that formation. This formation was chosen as it is a very representative formation; i.e., there is one agent along each axis of the LVLH frame. Hence, the performance of any agent, whose position is a linear combination of these known agent positions, can be approximated.

Figure 9: Reconfiguration from string-of-pearls formation to tetrahedron geometry in the LVLH frame (not drawn to scale)

Two main control architectures were designed for this particular mission. In the first controller, the dynamics of the CubeSat are assumed to be linear (in the LVLH frame) as the distance between the agents is relatively small. Various sensitivity analyses are carried out using this model as it helps us develop a good intuition for the system. Once a good understanding of the system is achieved, a nonlinear dynamic model is considered including $J_2$ effects. It is shown in the following sections that the results obtained from the two models are pretty close and a lot of the data obtained from the linear dynamics can be directly used.

1. Two-burn Controller with Linear Model

Since the distances between the CubeSats are relatively small, a linear model was chosen to describe the motion of the CubeSats. Hence, given the initial position and velocity of any agent, its position and velocity at time $t$ can be given by the solution of the HCW equations as given below.

$$
\begin{bmatrix}
  x(t) \\
y(t) \\
z(t) \\
\dot{x}(t) \\
\dot{y}(t) \\
\dot{z}(t)
\end{bmatrix}
= \begin{bmatrix}
  4 - 3 \cos \omega_z t & 0 & 0 & \sin \omega_z t / \omega_z & 2(1 - \cos \omega_z t) / \omega_z & 0 \\
  6 \sin \omega_z t - 6 \omega_z t & 1 & 0 & 2(-1 + \cos \omega_z t) / \omega_z & 4 \sin \omega_z t / \omega_z - 3t & 0 \\
  0 & 0 & \cos \omega_z t & 0 & 0 & \sin \omega_z t / \omega_z \\
  3 \omega_z \sin \omega_z t & 0 & 0 & \cos \omega_z t & 2 \sin \omega_z t & 0 \\
  6 \omega_z (-1 + \cos \omega_z t) & 0 & 0 & -2 \sin \omega_z t & -3 + 4 \cos \omega_z t & 0 \\
  0 & 0 & -\omega_z \sin \omega_z t & 0 & 0 & \cos \omega_z t
\end{bmatrix}
\begin{bmatrix}
x_0 \\
y_0 \\
z_0 \\
x_0 \\
y_0 \\
z_0
\end{bmatrix}
$$

(1)
Both the initial reconfiguration from the string-of-pearls to the tetrahedron formation as well as holding the formation in the tetrahedron shape was performed using a two-burn algorithm, i.e., the first thrust correction is applied to exit the current orbit and the second thrust correction is applied to enter the target or desired orbit. The reconfiguration time required to go from one orbit to another is inputted as a design choice. For the first burn, the initial and final positions of the agents are known, i.e., \( x_0, y_0, z_0, x(t), y(t) \) and \( z(t) \) are known. Since the reconfiguration time is also known, the set of equations given by Eq. (1) reduces to a set of 6 equations with 6 unknown velocities and hence can be solved for a unique solution.

Let \( \dot{x}_0-, \dot{y}_0- \) and \( \dot{z}_0- \) denote the velocity of the agent just before the thrust correction is applied and \( \dot{x}_0+, \dot{y}_0+ \) and \( \dot{z}_0+ \) denote the velocity of the agent just after the thrust correction is applied. Also, let \( \dot{x}(t), \dot{y}(t), \dot{z}(t) \) denote the velocity of the agent when it reaches its desired orbit. Then the velocity change required (\( \Delta V \)) for the two burns is specified by Eqs. (2−3). Equation 3 results from the fact that the entire velocity needs to be removed in the LVLH frame for the agent to align itself with the target orbit.

\[
\begin{align*}
\Delta V_1 &= \sqrt{(\dot{x}_0+ - \dot{x}_0-)^2 + (\dot{y}_0+ - \dot{y}_0-)^2 + (\dot{z}_0+ - \dot{z}_0-)^2} \\
\Delta V_2 &= \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2} \\
\Delta V &= \Delta V_1 + \Delta V_2 \\
\end{align*}
\]

Since the reconfiguration from a string-of-pearls configuration to a tetrahedron is only done once, Eqs. (2−4) can be directly applied with the initial and final positions specified. However, for the maintaining the formation as a tetrahedron, the thrusters have to be fired continuously as the agents have to be maintained in non-Keplerian orbits. Hence, the following control strategies were used to perform formation maintenance.

- Formation hold without threshold: Let the agent drift in any direction within an error ball, which is defined as a sphere with the vertex of the tetrahedron as its center. As soon as the agent drifts outside the sphere, apply the two-burn correction to bring it back to the center of the sphere with the first burn and enforce zero velocity at the center with the second burn. Let it drift again and keep correcting for the specified amount of time. This control strategy is depicted in Fig. 10a. This control strategy will imply less frequent burns (as compared to formation hold with threshold), but each burn will require more \( \Delta V \).

- Formation hold with threshold: Let the agent drift in any direction within an error ball, which is defined as a sphere with the vertex of the tetrahedron as its center. As soon as the agent drifts outside the sphere, apply the two-burn correction to bring it back to a distance from the center of the sphere (defined by the threshold) with the first burn and don’t enforce zero velocity at that point with the second burn. Let it drift again and keep correcting for the specified amount of time. This control strategy is depicted in Fig. 10b. This control strategy will imply more frequent burns (as compared to formation hold without threshold), but each burn will require less \( \Delta V \).

The values of the parameters used and the results obtained for a particular point design have been listed in Table 9 and Table 10 respectively. Since we do not have any particular requirements at this point, we decided to use some typical values for the mission parameters. The best position sensor currently available in the market (NovAtel GPS) gives a position error of approximately 1.2 meters per CubeSat. Hence, in the worst case scenario, the relative position error between CubeSats would be when both the individual errors are maximum, i.e., 2.4 meters. Hence, with a safety factor of 2, a value of 5 meters was chosen as the radius of the error ball. A sensitivity analysis was performed by varying the threshold value and it was observed that the minimum fuel is consumed when the threshold value is 0, i.e., the agent is kept at the edge of the error ball.

<table>
<thead>
<tr>
<th>Formation size</th>
<th>Formation altitude</th>
<th>Maintenance time</th>
<th>Error ball radius</th>
<th>Reconfiguration time</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 m</td>
<td>400 km</td>
<td>100 orbits</td>
<td>5 m</td>
<td>1 minute</td>
</tr>
</tbody>
</table>
Table 10: $\Delta V$ values for a point design using a two-burn controller on a linear model. CubeSat numbering is defined in Fig. 9.

<table>
<thead>
<tr>
<th></th>
<th>CubeSat 1 (m/s)</th>
<th>CubeSat 2 (m/s)</th>
<th>CubeSat 3 (m/s)</th>
<th>CubeSat 4 (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formation hold</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>without threshold</td>
<td>107.48</td>
<td>374.05</td>
<td>107.48</td>
<td>166.63</td>
</tr>
<tr>
<td>with threshold</td>
<td>6.96</td>
<td>96.78</td>
<td>6.96</td>
<td>17.06</td>
</tr>
</tbody>
</table>

Thus, the controller with threshold outperformed the controller without the threshold by a big margin. CubeSats 1 and 3 have identical values because they are symmetric according to the linear analysis. Also, we can see that using the current thruster technology (see Table. 6) it is not possible to achieve the $\Delta V$ required to hold the formation for 100 orbits. Hence, to understand the feasibility of various design points and also to understand the underlying trends, sensitivity analyses were carried out over the various parameter with the remaining parameters held at the above values.

Since CubeSat 2 in Fig. 9 is out of the plane of the base of the tetrahedron, the time period of its orbit will not match the time period of the other agents. This will result in a higher fuel consumption (as seen in Table 10) to maintain it at the given position because the agent will go outside its defined error ball faster than the others. Hence, to understand the variation of the fuel consumed or the $\Delta V$ required to hold CubeSat 2 in position, a sensitivity analysis was carried out by varying the height of CubeSat 2 for various formation sizes. The result of this analysis can be seen in Fig. 11.

It is clear from Fig 11 that the $\Delta V$ value increases as we increase the formation size. It can also be seen that the $\Delta V$ increases as the height of the CubeSat increases. Both these behaviours are expected because the orbits of the CubeSats are becoming more and more distinct and hence it would require larger amounts of fuel to keep them in place. We can see that the fuel required grows at a very high rate as the sizes increase. Since the plot shown gives the $\Delta V$ per orbit, the mission lifetime would be decided based on the chosen orbit parameters. For a typical lifetime of 100 orbits, the region of feasibility, given the current thruster technology, is restricted to height ratios smaller than 0.5 and formation sizes smaller than 200 meters. A possible way to improve the lifetime of the mission would be to periodically exchange CubeSat positions so that the fuel is consumed evenly amongst all CubeSats. However, since these may not be allowable for some mission types, such details have not been considered in this paper.
Figure 11: Variation of $\Delta V$ per orbit for CubeSat 2 with formation size and its height. Formation size refers to the side of the regular tetrahedron. The height ratio refers to the ratio of the current height of the CubeSat to the height of a regular tetrahedron of the given formation size. Negative height ratio implies that the CubeSat is below the plane of the base.

Another parameter of interest is the altitude of the formation in LEO which would vary depending on the mission requirements. Figure 12 shows the variation of $\Delta V$, with the size of the formation and the altitude of the formation, required to hold the formation in place for each of the CubeSats. CubeSat 3 has not been shown because it is identical to CubeSat 1.

Figure 12: Variation of $\Delta V$ per orbit for all the CubeSats with formation size and altitude of formation. Formation size refers to the side of the regular tetrahedron. Altitude refers to height of the formation above the surface of earth.
It can be seen from Fig. 12 that, as expected, the $\Delta V$ required for CubeSat 2 is the highest. Another interesting fact to note is that even though the $\Delta V$ increases with formation size, because atmospheric drag has not been incorporated into this model, the variation with altitude up to 1000 km is minimal. Hence, this factor can almost be neglected from the point of view of controller design. We prefer not to go above 1000 km altitude because not only the launch costs will increase but also the effects of solar radiation will become more noticeable and the CubeSat will start becoming more expensive if all the components need to have radiation shielding on them.

The accuracy with which an agent can hold its position is also an important factor. Scientific missions like synthetic aperture radar or radio interferomtry require high level of precision when it comes to position hold. On the other hand, applications like sampling the ionosphere can withstand a little more error without causing any significant effects on the measurements. Hence, the size of the allowed error ball in proportion to the size of the formation is an important factor to be considered. This variation is shown in Fig. 13. The behavior seen in Fig. 13 is slightly counter-intuitive because it appears that decreasing the size of the error ball results in a decrease in the the amount of $\Delta V$ required. Hence, it is less fuel consuming to hold a tighter formation in place. The reason for this behavior is that if we wait till the CubeSat goes farther away before applying the correction, the magnitude of the correction is higher. Hence, even though the frequency of correction would be lower, the magnitude of each correction is higher and because the magnitude changes faster than the frequency, the overall $\Delta V$ goes up.

![Figure 13: Variation of $\Delta V$ per orbit for all the CubeSats with formation size and size of error ball (Fig. 10a). Formation size refers to the side of the regular tetrahedron. Error ball refers to the distance the CubeSat is allowed to drift from its target location before the thrust correction is applied.](image-url)
between the edge of the sphere and the center. This was the motivation behind using the formation hold with threshold control strategy (Fig. 10b). A sensitivity analysis was carried out by varying the threshold and the reconfiguration time (both these variables are depicted in Fig. 10b). The results of this sensitivity analysis are shown in Fig. 14. It is clear from the plot that the $\Delta V$ increases as we make it go closer to the center. It can also be seen that the $\Delta V$ goes up as we decrease the reconfiguration time, which is the expected behavior. One important thing to note is that if we are keeping it closer to the edge of the error ball, the significance of the reconfiguration time starts to drop off, because it is going closer and closer to a continuous controller. Hence, for the linear dynamics, the optimal control strategy to reduce the $\Delta V$ is to use a near-continuous controller to keep the CubeSat at the edge of the error ball.

Figure 14: Variation of $\Delta V$ per orbit for all the CubeSats with threshold distance and reconfiguration time. Threshold distance refers to the distance that the CubeSat is forced to come back to after the correction is applied. Threshold distance of 0 would imply that the CubeSat is left on the edge of the error ball while a threshold distance of 5 implies that it is brought back to the center. Reconfiguration time refers to the time it is given to get back to that position.

2. Continuous Controller with Nonlinear Model with $J_2$ effects

From the previous section, it is clear that the better control strategy to use would be a continuous controller. However, the previous section used a linear model for the dynamics. There are many nonlinear factors which affect the dynamics of the CubeSat such as atmospheric drag, $J_2$ drift, solar radiation pressure etc. These factors can, sometimes, be non-trivial and might actually add a lot more constraints to the design. However, among all such nonlinearities, the one which causes the most deviation from linear dynamics is the $J_2$ drift term. Hence for the rest of this analysis, only the $J_2$ drift terms are considered and other effects such as drag, solar radiation pressure etc. are ignored. Hence, the equations of motion for the $j^{th}$ CubeSat in the LVLH frame considering $J_2$ perturbations are given below: $7, 67$
\[ \ddot{x}_j = 2\dot{y}_j \omega_z - x_j (\eta_j^2 - \omega_z^2) + y_j \alpha_z - z_j \omega_z \omega_x - (\zeta_j - \zeta) \sin i \sin \theta - r (\eta_j^2 - \eta^2) \]
\[ \ddot{y}_j = -2 \dot{x}_j \omega_z + 2 \dot{z}_j \omega_x - x_j \alpha_z - y_j (\eta_j^2 - \omega_z^2 - \omega_x^2) + z_j \alpha_x - (\zeta_j - \zeta) \sin i \cos \theta \]
\[ \ddot{z}_j = -2 \dot{y}_j \omega_x - x_j \omega_x \omega_z - y_j \alpha_x - z_j (\eta_j^2 - \omega_x^2) - (\zeta_j - \zeta) \cos i \]

where \( \omega_x, \omega_y, \omega_z \) are the rotation rates of the LVLH frame about the x, y and z axis respectively, \( \alpha_z \) is the orbital acceleration about the z axis, i.e., \( \omega_z \), \( J_2 \) is the second harmonic coefficient of Earth, \( \mu \) is the gravitational constant and \( R_e \) is radius of the earth and the terms \( \eta, \eta_j, \zeta, \zeta_j, r_j, \) and \( r_{jZ} \) have been introduced in order to simplify the potential energy terms as shown below:

\[ k_{j2} = \frac{3}{2} J_2 \mu R_e^2, \quad \zeta = \frac{2k_{j2} \sin i \sin \theta}{r^4} \]
\[ \zeta_j = \frac{2k_{j2} r_{jZ}}{r^5} \]
\[ \eta^2 = \frac{\mu}{r^3} + \frac{k_{j2}}{r^5} - \frac{5k_{j2} \sin^2 i \sin^2 \theta}{r^5} \]
\[ \eta_j^2 = \frac{\mu}{r_j^3} + \frac{k_{j2}}{r_j^5} - \frac{5k_{j2} r_{jZ}^2}{r_j^5} \]
\[ r_j = \sqrt{(r + x_j)^2 + y_j^2 + z_j^2} \]
\[ r_{jZ} = (r + x_j) \sin i \sin \theta + y_j \sin i \cos \theta + z_j \cos i \]

and the chief orbital motion of the chief spacecraft (or the virtual chief orbit) is derived by Gauss’s variational equations’ as given below:

\[ \dot{\alpha} = f(\alpha) \]

where the orbital element vector \( \alpha \) could use a hybrid representation such as \( \alpha = (r, v_x, h, \Omega, i, \theta) \) where \( r, v_x, h, \theta, i, \) and \( \Omega \) denote the geocentric distance, the radial velocity, the angular momentum magnitude, the argument of latitude, the inclination, and the longitude of the ascending node respectively.

Given these dynamic equations, a PID controller was designed to hold a formation about a desired point. Since the inclination of the orbit will also make a difference in the results, we considered the two extreme cases for it, i.e, \( 0^\circ \) inclination and \( 45^\circ \) inclination. The gains are tuned such that the maximum error in the position is always less than 5 meters, which is the same as that of the two-burn controller. The results are summarized in the Table 11.

Table 11: \( \Delta V \) values for a point design using a two-burn controller on a linear model. CubeSats have the same numbering as defined in Fig 9.

<table>
<thead>
<tr>
<th>CubeSat</th>
<th>CubeSat 1 (m/s)</th>
<th>CubeSat 2 (m/s)</th>
<th>CubeSat 3 (m/s)</th>
<th>CubeSat 4 (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit Inclination (i)</td>
<td>0(^\circ)</td>
<td>45(^\circ)</td>
<td>0(^\circ)</td>
<td>45(^\circ)</td>
</tr>
<tr>
<td>Continuous controller</td>
<td>42.47</td>
<td>23.35</td>
<td>177.96</td>
<td>93.15</td>
</tr>
</tbody>
</table>

Comparing the values given in Table 11 to the values given in Table 10, we can see that the continuous controller always does better than the two-burn controller with no threshold. However, the two-burn controller with threshold does seem to do better than the continuous controller. This difference can be attributed to the nonlinear effects of \( J_2 \) perturbations, as these will cause the controller to burn more fuel.

To compare the results of the continuous controller with the two-burn controller, we also conducted similar trade studies that we performed with the two-burn method on the continuous controller. Fig 15 shows the sensitivity analysis of the size of the formation against its altitude using the continuous controller. We can see that the trends seen is the same as the two-burn controller (Fig. 12).
Figure 15: Variation of $\Delta V$ per orbit for all the CubeSats with formation size and altitude of formation using continuous controller. Formation size refers to the side of the regular tetrahedron. Altitude refers to height of the formation above the surface of earth.

3. Initial time difference between launches

In the previous two subsections, we assumed that the CubeSats are initially at a distance of 1 km away from each other. However, in reality, multiple 3U CubeSats cannot fit inside the same PPOD and hence, there would be a time difference between the launches of two CubeSats. The CubeSats would hence be separated by a distance greater than 1 km and therefore, some amount of fuel would need to be burnt in order to bring them closer to each other.

Without lose of generality, let us consider the case of two CubeSats launched from the same launch vehicle (i.e. same point in space) with some time difference between their launches. Since the actual time difference between multiple launches is highly mission specific and no data is available on the same, an initial angular separation of $\pi/10$ radians is assumed between the two CubeSats. Let us also assume that the two CubeSats are initialized on circular orbits with zero inclination at an altitude of 400 km. After 1 orbit (can be done instantly as well), the leading CubeSat is given a tangential $\Delta V$ of 10 m/s in order to slightly alter its orbit and make it elliptical. The two CubeSat orbits are then propagated in time. The results of this simulation are shown in Fig. 16. We can clearly see that the CubeSats are within close proximity of each other within 12 orbits (approximately 1 day). Thus, even if they are far away from each other at the time of launch, the CubeSats can be brought closer together without using too much fuel. The fuel used can further be decreased if the longer reconfiguration periods are tolerable. This $\Delta V$ can also be shared between the two CubeSats if both of them apply their thrusters in opposite directions. Similar calculations can also be performed for multiple CubeSats in different initial configurations.
Figure 16: Initial reconfiguration of two CubeSats to correct time difference between their launches. The two CubeSats are represented by the red and black circles and their orbits are represented by the same color.

B. Reconfiguring among multiple $J_2$ invariant relative orbits

The discussion of the previous section dealt with holding the CubeSats in a constant formation. However, it was seen that the fuel required for such a mission is very high and hence with the current thrusters, only mission lives of the order on one month can be achieved. If we want to extend the mission life, this kind of active formation control is not feasible. On the other hand, if we have the CubeSats which are in some form of relative orbits such that after certain time intervals, they seem to appear in the same formation, then the amount of fuel required to keep them in that passive formation is very low. $J_2$ invariant orbits have been shown to provide collision free motion for swarms of satellites in low Earth orbits.\(^7\) We can also periodically switch between multiple such $J_2$ invariant orbits from time to time and then maintain that orbit if a different passive formation is desired. However, while reconfiguring among such orbits, one needs to ensure that there are no collisions among the CubeSats and also that the reconfigurations are done in an optimal fashion in order to reduce fuel consumption. Model predictive control using sequential convex programming (MPC-SCP)\(^68\) was used as a tool for such reconfigurations as it guarantees fuel optimal trajectories while ensuring collision avoidance.

In this simulation, 6 agents were reconfigured between randomly selected initial positions with a spread of 500 meters as shown in Fig. 17. The chosen orbit was a 400 km circular orbit with an inclination of 45°.

In order to validate the MATLAB simulations that were carried out in Ref. 68, the results were compared using a commercially available software package named System Tool Kit (STK)\(^69\) from Analytical Graphics Inc. Both the dynamics were propagated for 500 orbits using the initial $J_2$-invariance condition\(^68\) and the drift in the x, y and z values of the agent from its original trajectory (in the LVLH frame) were compared.
Figure 17: Reconfiguration of 6 CubeSats among different $J_2$ invariant orbits using model predictive control (not drawn to scale).

The input parameters that were used by the simulations are:

<table>
<thead>
<tr>
<th>Semi-major axis</th>
<th>Eccentricity</th>
<th>Inclination</th>
<th>Longitude of the ascending node</th>
<th>Argument of periapsis</th>
</tr>
</thead>
<tbody>
<tr>
<td>6878 km</td>
<td>0.001</td>
<td>45°</td>
<td>0°</td>
<td>0°</td>
</tr>
</tbody>
</table>

It was seen that the mean drift along the x-axis of the LVLH is 0 and this is consistent because the CubeSat cannot gain energy and hence cannot change its altitude. Fig. 18 shows the difference in the drifts, in the y and z axis, between the MATLAB and the STK simulations. The time axis extends till 500 orbits. It can be seen that the difference in the errors is very small and in the order of millimeters even after 500 orbits. Even these minute differences between the STK and the MATLAB results can be attributed to errors such as numerical integration errors, precision of the physical parameter values etc. The total absolute drift after 500 orbits along the z-axis was around 73 mm (0.146 mm/orbit), and along the y axis was around 2.8 meters (5.6 mm/orbit). These errors are very small and can be easily corrected at a very low fuel cost.

Since sequential convex programming can also simulate sensor and actuator errors, actual error quotes taken from hardware were integrated into the algorithm. The thruster chosen was the Nanosatellite Micro-
propulsion System (ISIS)\textsuperscript{17} which has a maximum $\Delta V$ of 100 m/s. The position and velocity were obtained using the NovAtel GPS.\textsuperscript{39} Also, since model predictive control is completely decentralized\textsuperscript{68} each agent only needs to communicate with its neighbors during the reconfiguration. Thus for communication purposes, the Xbee Pro RF\textsuperscript{60} communication chip was chosen. Fig. 19 shows the $\Delta V$ consumed as a function of the number of reconfigurations for each of the 6 CubeSats.

Hence, we can reconfigure up to 20 times using the current thruster technology. It should be noted that this does not restrict the mission life as the CubeSats can stay in any $J_2$ invariant orbit for a very long time while burning minimal amounts of fuel. This will only restrict the number of distinct passive formations that can be achieved.

V. Cubesat Relative Positioning

As relative sensing was identified as another major bottleneck in the design, further analyses has been carried out in this section. As shown in Section III, NovAtel receiver was identified as the best (currently available) GPS that is available off-the-shelf. Even with reduced atmospheric errors, a maximum accuracy of only two meters can be obtained by code-based navigation solutions. For inter-satellite distance measurement, this level of ranging is not accurate enough for precise formation flying. In order to achieve centimeter level accuracy, differential GPS (DGPS) and carrier phase measurements are used.

DGPS and carrier phase based positioning are mature technologies that are commonly used in static and kinematic precise positioning operations in terrestrial environments. The use of DGPS has been extended for use in a few cooperative spacecraft missions such as the NASA GRACE mission where the satellites have the processing power and budget for highly precise GPS receivers.\textsuperscript{70} However it is only coming into emergence in the CubeSat space. With the emergence of CubeSats as a platform for cooperative formation flying, the use of DGPS is expected to increase. One current upcoming example is the CanX-4 & 5 missions where DGPS is used to locate and aid in control of formation flying CubeSats.\textsuperscript{71} The use of DGPS leverages an existing common sensor for relative ranging without the need of additional payload onto the limited carrying capacity of a CubeSat. Because the goal of formation flying CubeSat missions typically consists of more than two CubeSats, implementing a custom DGPS solution may provide more accurate relative ranging, since off-the-shelf DGPS solutions are typically designed to be between two entities, the user and reference receivers.
A. Differential GPS

The GPS L1 carrier frequency is 1575.42 MHz, which in turn specifies a wavelength of 0.1904 meters. Carrier phase measurements are modelled by the following equation:

$$\phi(t) = \frac{1}{\lambda} [r(t) - I + T] + f(\delta t_u - \delta t_s) + N - \epsilon_\phi \tag{9}$$

where $\phi$ is the partial phase cycles measured by the receiver, $\lambda$ is the L1 signal wavelength at 0.1904 meters, $r(t)$ is the true distance between the GPS satellite and the receiver, $I$ and $T$ are the ionospheric advance and tropospheric delay respectively, representing the atmospheric errors, $f$ is the L1 frequency at 1575.42 MHz, $\delta t_u$ and $\delta t_s$ are the receiver and satellite clock biases respectively, $N$ is the integer ambiguity term or the full phase cycles that have passed through the transmission between the satellite and receiver and $\epsilon_\phi$ is the carrier phase measurement error.

By taking advantage of the carrier phase measurements and the geometry between each receiver and the satellites in view, the range or baseline between the receivers can be determined. This is accomplished through differencing otherwise known as Differential GPS (DGPS).

One satellite is set as the user receiver while the other is set as the reference receiver. Each receiver takes phase measurements to the same set of satellites.

$$r_{ur}^k(t) = r_u^k(t) - r_r^k(t) = (-1)^k \cdot x_{ur} \tag{10}$$

Fig. 20 shows the geometry differencing that makes baseline determination possible. The distance vector between the two receivers, $r_{ur}^k$, can be formed from the difference of the each receiver to CubeSat k’s distances and the unit vectors to each kth CubeSat and is given by.

$$\phi_{ur}^k(t) = \phi_u^k(t) - \phi_r^k(t) \tag{11}$$

$$\phi_{ur}^k(t) = \frac{1}{\lambda} [r_{ur}^k(t) - I_{ur}^k - (I_{u}^k(t) - I_{r}^k) + (T_{u}^k(t) - T_{r}^k)] + f(\delta t_u - \delta t_r)$$

$$+ (N_u^k - N_r^k) + (\epsilon_{\phi,u}^k - \epsilon_{\phi,r}^k) \tag{12}$$

$$\phi_{ur}^k(t) = \frac{1}{\lambda} [r_{ur}^k - I_{ur}^k + T_{ur}^k] + f\delta t_{ur} + N_{ur}^k + \epsilon_{\phi,ur}^k \tag{13}$$

For a set of K GPS satellites, a system of linear equations can be formed as shown below.
In order to take advantage of integer ambiguities, the single difference equations are further differenced into double differencing as given below.\textsuperscript{73}

\[
\begin{bmatrix}
\phi_{ur}^1 \\
\phi_{ur}^2 \\
\phi_{ur}^3 \\
\phi_{ur}^{K1}
\end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix}
\cdot \\
\cdot \\
\cdot \\
\cdot 
\end{bmatrix} \begin{bmatrix} (1^2 - 1^1)^T \\
(1^3 - 1^1)^T \\
\cdot \\
\cdot 
\end{bmatrix} x_{ur} + f \delta t_{ur} \begin{bmatrix} 1 \\
1 \\
1 \\
1 
\end{bmatrix} + \begin{bmatrix} N_{ur}^1 \\
N_{ur}^2 \\
\cdot \\
N_{ur}^{K1} 
\end{bmatrix} + \begin{bmatrix} \epsilon_{\phi,ur}^1 \\
\epsilon_{\phi,ur}^2 \\
\cdot \\
\cdot 
\end{bmatrix}
\]

In order to take advantage of integer ambiguities, the single difference equations are further differenced into double differencing as given below.\textsuperscript{73}

\[
\begin{align*}
\phi_{ur}^{kl} &= (\phi_{u}^k - \phi_{r}^l) - (\phi_{u}^l - \phi_{r}^l) \\
\phi_{ur}^{kl} &= \phi_{ur}^k - \phi_{ur}^l \\
\phi_{ur}^{kl} &= \frac{1}{\lambda} (r_{ur}^k - r_{ur}^l) + f (\delta t_{ur} - \delta t_{ur}) + (N_{ur}^k - N_{ur}^l) + (\epsilon_{ur}^k - \epsilon_{ur}^l) \\
\phi_{ur}^{kl} &= \frac{1}{\lambda} r_{ur}^{kl} + N_{ur}^{kl} + \epsilon_{ur}^{kl} \\
\rho_{u}^{kl} &= r_{u}^{kl} - r_{r}^{kl} - (r_{u}^l - r_{r}^l) \\
\rho_{u}^{kl} &= -1^{l} r_{u}^{kl} + 1^{r} r_{u}^{kl} \\
\rho_{u}^{kl} &= -(1^{k} - 1^{l}) x_{ur}
\end{align*}
\]

\[
\begin{bmatrix}
\phi_{ur}^{31} \\
\phi_{ur}^{31} \\
\phi_{ur}^{K1} \\
\phi_{ur}^{K1}
\end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix}
\cdot \\
\cdot \\
\cdot \\
\cdot 
\end{bmatrix} \begin{bmatrix} (1^2 - 1^1)^T \\
(1^3 - 1^1)^T \\
\cdot \\
\cdot 
\end{bmatrix} x_{ur} + \begin{bmatrix} 1 \\
1 \\
1 \\
1 
\end{bmatrix} + \begin{bmatrix} N_{ur}^1 \\
N_{ur}^2 \\
\cdot \\
N_{ur}^{K1} 
\end{bmatrix} + \begin{bmatrix} \epsilon_{\phi,ur}^{31} \\
\epsilon_{\phi,ur}^{31} \\
\cdot \\
\cdot 
\end{bmatrix}
\]

B. Integer Ambiguity Resolution

The integer ambiguities must be resolved in order to solve the differencing equations for the baseline vector. Integer ambiguity resolution is usually resolved through different search methods. One approach is the multiple epoch method where observations of the satellites are taken at multiple epochs spaced apart enough for the satellite constellation to move, typically separated by thirty minutes. This is one of the least complex methods of solving for the ambiguities; however, the trade-off is the time needed for integer ambiguity resolution. In a CubeSat application, for real-time control, this method is highly ineffective because there is a large initialization time before CubeSat control may be accomplished.

For real-time integer ambiguity resolution, different strategies are applied in order to reduce the search space of possible integer ambiguities. Typically a float solution or initial estimate is found for the integer ambiguities. Afterwards a search algorithm is used to identify the likely values. Then optimization methods are used to select the best values found through the search algorithm. The values are then substituted into the double differencing equations in the previous section for baseline determination. If a cycle slip, or loss of tracking of the carrier phase cycles occurs on a selected channel, integer ambiguity resolution must be performed again for that specific satellite.

In order to find a float solution, the noisy but unambiguous pseudorange measurements can be smoothed by the more precise carrier phase measurements in a process called carrier smoothing. This can be accomplished through a Hatch Filter.\textsuperscript{72}

\[
\rho_{hat}(t) = \frac{1}{n} \rho(t) + \frac{(n - 1)}{n} \left[ \rho_{hat}(t - 1) + (\phi(t) - \phi(t - 1)) \right]
\]

where $\rho$ is the pseudo range and $\rho_{hat}(t)$ is the carrier smoothed pseudo range.

Fig. 21 shows a comparison of the double difference errors between using the carrier phase measurements and the noisier code measurements. By applying the carrier phase measurements and using them to smooth
the pseudoranges, initial errors can be constrained to centimeter level accuracy. By using smoothed pseudo-
ranges, the search space of each integer ambiguity can be reduced to 5-10 wavelengths and each ambiguity
difference to 10-20 wavelengths.

Different search techniques can then be used to effectively search through the resultant search space. One
approach is the least-squares ambiguity decorrelation adjustment (LAMBDA) method\textsuperscript{74}, which decorrelates
the ambiguities and reshapes the search space. The resultant covariances of the resultant float solution is
an ellipsoid and is difficult to search over. The LAMBDA method reshapes the ellipsoid into a more spherical
search space, resulting a much more efficient search.

C. Experimental Validation

A ground based experiment was set up to verify the ranging algorithm. Two low-cost ublox LEA-6T GPS
receivers\textsuperscript{75} were set at a measured distance apart for close range precision ranging.

Both pseudorange and carrier phase measurements were taken from both receivers and then combined in
post processing for relative range calculation. The receivers were placed at a distance of one meter apart in
order to have a ground truth to compare the relative ranging errors against.

Fig. 23 shows the comparison of error between Code Measured GPS and DGPS relative ranging error
between the two receivers. As shown in the figure, DGPS is able to obtain a much more accurate ranging
solution with an error of less than 20 cm.
From its terrestrial applications as well as our experiment, it has been shown that DGPS can be a cost-effective way of adding accurate relative ranging to CubeSats. However it is essential for the Cube-Sats to be equipped with carrier measurement capable receiver resistant to carrier cycle slips, or loss of carrier-measurement-tracking. This prevents the need to rerun the computationally intensive integer ambiguity resolution and the temporary ranging loss that may occur during retracking. Carrier smoothing of pseudorange measurements can already improve positioning and relative ranging of CubeSats to sub-meter accuracies. If tighter accuracies are required to hold smaller formations in place, then carrier based DGPS can be used to improve ranging to centimeter accuracies.

VI. Conclusions

After an extensive literature survey of CubeSat missions and component technologies, it was concluded that we stand to gain a lot by using multiple small satellites instead of a single monolithic satellite. Formation flying using multiple small satellites also has many important scientific applications. However, the technologies in some fields are still a little premature and need some development before we can actually launch formations of small satellites with scientific payloads. This paper provides the general areas in which research and development is needed with respect to CubeSat technologies. In particular thrusters (or position actuators) and relative position sensing are the major technological bottlenecks preventing active formation control for missions lasting longer than a month. To characterize the abilities of currently available thrusters, detailed simulations were run over a wide range of mission design parameters using multiple control strategies for a representative 3D formation. It was shown that the chosen thruster must be capable of a $\Delta V$ of at least 100 m/s for the formation to maintain a 5 meter position accuracy for 100 orbits. It was also shown that upto 20 reconfigurations can be performed between multiple $J_2$ invariant relative orbits. To improve current relative sensing options, we employed differential GPS to improve the accuracy of the receivers from 2 meters to 20 centimeters. The results were verified in a hardware test using two low-cost u-blox receivers placed 1 meter apart. On the basis of our simulations and tests, we believe that the best (current) actuators and sensors for our mission design are:

- Thruster: Nanosatellite Micropropulsion System from ISIS
- Position Determination: NovAtel OEM615 GPS
- Attitude Determination and Control System: XACT from Blue Canyon Tech
- Communication: XBee Pro
We believe that the other subsystems which have not been mentioned above like power, onboard computers, structures etc. have already reached the required level of maturity and hence components that are directly available of the shelf can be used for these applications.

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